AQA Mechanics

Topic Questions from Papers

Kinematics

Answers

1 (a)(i)	$a = 2 + 12e^{-t}$	M1A1	2	Differentiating, with at least one term correct. Correct velocity
(ii)	$2 < a \leq 14$	B1,B1 B1	3	For 2, For 14 Correct inequalities
(b)	$s = t^2 + 12e^{-t} + c$	M1 A1		Integrating, with at least one term correct. Correct expression with or without c
	$s = 0, t = 0 \Rightarrow c = -12$	dM1		Finding c
	$s = t^2 + 12e^{-t} - 12$	A1	4	Correct final expression
	Total		**9**	

(Q3, Jan 2006)

2 (a)	$\mathbf{v} = (6t^2 - 2t)\mathbf{i} + (1 - 12t^2)\mathbf{j}$	M1 A1 A1	3	differentiating both components one component correct second component correct
(b)(i)	$\mathbf{v}\left(\frac{1}{3}\right) = \left(\frac{6}{9} - \frac{2}{3}\right)\mathbf{i} + \left(1 - \frac{12}{9}\right)\mathbf{j} = -\frac{1}{3}\mathbf{j}$	M1 A1	2	substituting the value for t into their $\mathbf{v}$ correct velocity
(ii)	Travelling due south	A1ft	1	correct description (Follow through from $\mathbf{v} = \pm k\mathbf{j}$)
(c)	$\mathbf{a} = (12t - 2)\mathbf{i} - 24t\mathbf{j}$ $\mathbf{a}(4) = 46\mathbf{i} - 96\mathbf{j}$	M1 A1 A1	3	differentiating their velocity correct acceleration at time t correct acceleration at $t = 4$
(d)	$\mathbf{F} = 6(46\mathbf{i} - 96\mathbf{j}) = 276\mathbf{i} - 576\mathbf{j}$	M1		apply Newton's second law correctly
	$F = \sqrt{276^2 + 576^2} = 639$ N or $a = \sqrt{46^2 + 96^2} = 106.45$ $F = 6 \times 106.45 = 639$ N	M1 A1	3	finding magnitude correct magnitude
	Total		**12**	

(Q1, June 2006)

3 (a)(i)	$t = 0,\ \mathbf{r} = 2\mathbf{i} + 10\mathbf{k}$	B1	1	
(ii)	$t = 2\pi,\ \mathbf{r} = 2\mathbf{i} + 7.49\mathbf{k}$	B1	1	Or $\mathbf{r} = 2\mathrm{i} + (10 - 0.8\pi)\mathbf{k}$ accept $7.5\mathbf{k}$
(iii)	$t = 2\pi, \qquad t = 4\pi$	B1 B1	2	
(b)	$\mathbf{v} = -2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} - 0.4\mathbf{k}$	M1 A1 A1	3	Differentiation Trig **k**
(c)	$\mathbf{a} = -2\cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$ $\mathbf{F} = -50\cos t\,\mathbf{i} - 50\sin t\,\mathbf{j}$ $\lvert\mathbf{F}\rvert = \sqrt{50^2\cos^2 t + 50^2\sin^2 t}$ $\lvert\mathbf{F}\rvert = 50(\mathrm{N})$	M1A1 M1 M1 A1	5	 No unit vectors
	Total		**12**	

(Q5, Jan 2007)

4 (a)	Using $F = ma$: $2400\mathbf{i} - 4800t\mathbf{j} = 800\mathbf{a}$ $\mathbf{a} = 3\mathbf{i} - 6t\mathbf{j}$	 M1 A1	 2	
(b)	$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$ $= 3t\mathbf{i} - 3t^2\mathbf{j} + \mathbf{c}$	M1 A1		 Condone no '+ **c**'
	When $t = 0,\ \mathbf{v} = 6\mathbf{i} + 30\mathbf{j}$ $\therefore\ \mathbf{c} = 6\mathbf{i} + 30\mathbf{j}$ $\therefore\ \mathbf{v} = (3t + 6)\mathbf{i} + (30 - 3t^2)\mathbf{j}$	 M1 A1	 4	 Needs '+ **c**' above AG
(c)	$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$ $= (\frac{3}{2}t^2 + 6t)\mathbf{i} + (30t - t^3)\mathbf{j} + \mathbf{d}$	M1 A1,A1		 A1 **i** term, A1 **j** term; condone no '+ **d**'
	When $t = 0,\ \mathbf{r} = 2\mathbf{i} + 5\mathbf{j}$ $\therefore\ \mathbf{d} = 2\mathbf{i} + 5\mathbf{j}$ $\therefore\ \mathbf{r} = (\frac{3}{2}t^2 + 6t + 2)\mathbf{i} + (30t - t^3 + 5)\mathbf{j}$	 M1 A1	 5	
	Total		**11**	

(Q3, June 2007)

5 (a)(i)	$a = \frac{dv}{dt} = 6t - 6\cos 3t$	M1A1	2	M1 for at least one term correct
(ii)	When $t = \frac{\pi}{3}$, $a = 6 \times \frac{\pi}{3} - 6\cos(3.\frac{\pi}{3})$	M1		
	$= 2\pi + 6$	A1	2	AG
(b)	$r = t^3 + \frac{2}{3}\cos 3t + 6t + c$	M1A1		M1 for 3 terms including $\cos 3t$ term Condone no '+ c'
	When $t = 0$, $r = 0$ $\therefore c = -\frac{2}{3}$	M1		
	$\therefore r = t^3 + \frac{2}{3}\cos 3t + 6t - \frac{2}{3}$	A1	4	
	Total		**8**	

(Q2, Jan 2008)

6 (a)	$\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = (3t^2 - 6t)\mathbf{i} + (4 + 2t)\mathbf{j}$	M1A1	2	
(b)(i)	$\mathbf{a} = (6t - 6)\mathbf{i} + 2\mathbf{j}$	M1 A1ft		
	Using $\mathbf{F} = m\mathbf{a}$: $\mathbf{F} = (18t - 18)\mathbf{i} + 6\mathbf{j}$	A1ft	3	
(ii)	When $t = 3$, $\mathbf{F} = 36\mathbf{i} + 6\mathbf{j}$ Magnitude is $\sqrt{36^2 + 6^2}$	M1		
	$= 36.5$	A1ft	2	Accept $6\sqrt{37}$; ft from (b)(i)
(c)	When **F** acts due north: Component of **F** in the **i** direction is 0 $18t - 18 = 0$	M1		
	$t = 1$	A1ft	2	ft from (b)(i)
	Total		**9**	

(Q4, Jan 2008)

7 (a)	$a = \frac{dv}{dt} = 12t + 4$	M1 A1	2	
(b)	Using $F = ma$, Force $= 3 \times (12t + 4)$ When $t = 4$, force $= 3\ (12 \times 4 + 4)$	M1		
	Force $= 156$ N	A1	2	
(c)	$r = 2t^3 + 2t^2 - 7t + c$	M1 A1		
	When $t = 0$, $r = 5$, $\therefore c = 5$	M1		
	$\therefore r = 2t^3 + 2t^2 - 7t + 5$	A1	4	SC3 if no '+c' seen
	Total		**8**	

(Q1, June 2008)

8	$r = \int v\,\mathrm{d}t$	M1		
	$= t^4 + 4\cos 2t + 5t\ (+c)$	A1		
	When $t = 0$, $r = 0 \Rightarrow c = -4$	M1		Finding c correctly
	$\therefore r = t^4 + 4\cos 2t + 5t - 4$	A1ft	4	
	Total		**4**	

(Q1, Jan 2009)

9 (a)	$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	M1		
		A1		**i** terms
	$\mathbf{v} = (\mathrm{e}^{\frac{1}{2}t} - 8)\mathbf{i} + (2t - 6)\mathbf{j}$	A1	3	**j** terms
(b)(i)	When $t = 3$, $\mathbf{v} = -3.52\mathbf{i}$	B1		Accept $(\mathrm{e}^{\frac{3}{2}} - 8)\mathbf{i}$
	Speed is 3.52 m s $^{-1}$	B1	2	3.5 does not give 2nd B mark
(ii)	West	B1	1	
(c)	$\mathbf{a} = \frac{1}{2}\mathrm{e}^{\frac{1}{2}t}\mathbf{i} + 2\mathbf{j}$	M1A1		
	When $t = 3$, $\mathbf{a} = \frac{1}{2}\mathrm{e}^{\frac{3}{2}}\mathbf{i} + 2\mathbf{j}$ or $2.24\mathbf{i} + 2\mathbf{j}$	A1	3	
(d)	Using $\mathbf{F} = m\mathbf{a}$:	M1		Accept $\mathbf{F} = 7\mathbf{a}$
	$\mathbf{F} = 7(\frac{1}{2}\mathrm{e}^{\frac{3}{2}}\mathbf{i} + 2\mathbf{j})$			
	$\therefore$ Magnitude of force is			
	$7\left((\frac{1}{2}\mathrm{e}^{\frac{3}{2}})^2 + 2^2\right)^{\frac{1}{2}}$	M1		
	$\mathbf{F} = 21.025$			
	$\mathbf{F} = 21.0$	A1	3	Accept 21
	Total		**12**	

(Q3, Jan 2009)

10 (a)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = (3t^2 - 15)\mathbf{i} + (6 - 2t)\mathbf{j}$	M1A1 A1	3	A1 (**i** terms) A1 (**j** terms)
(b)(i)	Using $\mathbf{F} = m\mathbf{a}$:			
	Force $= 4 \times \{(3t^2 - 15)\mathbf{i} + (6 - 2t)\mathbf{j}\}$	M1		
	$= (12t^2 - 60)\mathbf{i} + (24 - 8t)\mathbf{j}$	A1	2	AG
(ii)	When $t = 2$, force $= -12\mathbf{i} + 8\mathbf{j}$	M1A1		
	Magnitude of force $= \sqrt{12^2 + 8^2}$ N	M1		
	$= 14.4$ (N)	A1	4	
	Total		**9**	

(Q1, June 2009)

11 (a)	$\mathbf{r} = \int \mathbf{v}\,dt$ $= \left(t^4 - 6t^2 + 3t\right)\mathbf{i} + 5t\mathbf{j} + 4t^2\mathbf{k} + \mathbf{c}$ When $t = 0$, $\mathbf{r} = -5\mathbf{i} + 6\mathbf{k}$ $\therefore \mathbf{c} = -5\mathbf{i} + 6\mathbf{k}$ $\therefore \mathbf{r} = \left(t^4 - 6t^2 + 3t - 5\right)\mathbf{i} + 5t\mathbf{j} + \left(6 + 4t^2\right)\mathbf{k}$	M1 A1m1 A1	 4	M1 for at least one term correct m1 for $+ \mathbf{c}$
(b)	$\mathbf{a} = \left(12t^2 - 12\right)\mathbf{i} + 8\mathbf{k}$	M1A1	2	M1 for either component
(c)	Magnitude is $\left\{\left(12t^2 - 12\right)^2 + 64\right\}^{\frac{1}{2}}$	M1 A1F	2	
(d)	Magnitude is a minimum when $12t^2 - 12$ is zero ie when $t =$ 1	M1 A1	2	M1 for correct differentiation of correct expression in (c)
(e)	Minimum acceleration is 8 Using F = *ma*, $F = 7 \times 8 = 56$	 M1 A1	 2	 *a* could be a vector CAO
	Total		**12**	

(Q4, Jan 2010)

12	$v = \frac{ds}{dt}$ $= 10t - 12\sin 4t$	M1 A1A1	 3	M1 for either $\frac{ds}{dt}$ or 1 of 2 terms correct (ignore signs)
	Total		**3**	

(Q1, June 2010)

13 (a)	Using **F** = *m***a**,			
	$400\cos\frac{\pi}{2}t\,\mathbf{i} + 600t^2\,\mathbf{j} = 200\,\mathbf{a}$	M1		
	$\mathbf{a} = 2\cos\frac{\pi}{2}t\,\mathbf{i} + 3t^2\,\mathbf{j}$	A1	2	
(b)	$\mathbf{v} = \int a\ \mathrm{d}t$	M1		M1 for either $\int a\ \mathrm{d}t$ or 1 of 2 terms correct
	$= \frac{4}{\pi}\sin\frac{\pi}{2}t\,\mathbf{i} + t^3\mathbf{j} + \mathbf{c}$	A1m1		m1 for + **c**
	When $t = 4$, $\mathbf{r} = -3\mathbf{i} + 56\mathbf{j}$,			
	$64\mathbf{j} + \mathbf{c} = -3\mathbf{i} + 56\mathbf{j}$	m1		
	$\therefore \mathbf{c} = -3\mathbf{i} - 8\mathbf{j}$			
	$\therefore \mathbf{v} = (\frac{4}{\pi}\sin\frac{\pi}{2}t - 3)\mathbf{i} + (t^3 - 8)\mathbf{j}$	A1	5	Do not accept $\frac{2}{\frac{\pi}{2}}$ Accept 1.27 for $\frac{4}{\pi}$
(c)	When particle is moving due west, northerly component is zero	M1		
	$\therefore t^3 - 8 = 0$	A1√		
	$t = 2$	A1	3	
(d)	When $t = 2$, $\mathbf{v} = -3\mathbf{i} + 0\mathbf{j}$	B1√		
	Speed of particle is 3 m s^{-1}	B1	2	B1 for change –3 to +3
	Total		**12**	

(Q4, June 2010)

14 (a)	$\mathbf{r} = \int v\ \mathrm{d}t$			
	$= (4t + t^3)\mathbf{i} + (12t - 4t^2)\mathbf{j} + \mathbf{c}$	M1A1		M1 either **i** or **j** term correct. Condone no **c**
	When $t = 0$, $\mathbf{r} = 5\mathbf{i} - 7\mathbf{j}$			
	$\mathbf{c} = 5\mathbf{i} - 7\mathbf{j}$	M1		Any attempt at **c**
	$\mathbf{r} = (5 + 4t + t^3)\mathbf{i} + (-7 + 12t - 4t^2)\mathbf{j}$	A1	4	
(b)	$\mathbf{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$			
	$\mathbf{a} = 6t\,\mathbf{i} - 8\,\mathbf{j}$	M1A1	2	M1 either term correct
(c)	Using **F** = *m***a**	M1		Or: using **F** = *m***a**
	$\mathbf{F} = 2(6t\,\mathbf{i} - 8\,\mathbf{j})$			$\mathbf{F} = 2(6t\,\mathbf{i} - 8\,\mathbf{j})$
	$= 12t\,\mathbf{i} - 16\,\mathbf{j}$	A1		When $t = 1$, $\mathbf{F} = 12\,\mathbf{i} - 16\,\mathbf{j}$
	$\therefore$ Magnitude of force is			
	$(144t^2 + 256)^{\frac{1}{2}}$ when $t = 1$	M1		Magnitude of force is $(144 + 256)^{\frac{1}{2}}$
	= 20 N	A1	4	= 20 N
	Total		**10**	

(Q1, Jan 2011)

15 (a)	$\mathbf{a} = \frac{dv}{dt}$ $\mathbf{a} = -8e^{-2t}\mathbf{i} + (6-6t)\mathbf{j}$	M1 A1 A1	3	M1: Differentiating with either of the two components correct. Do not need to see **i** or **j**. A1: Correct **i** component. A1: Correct **j** component.
(b)(i)	Using $\mathbf{F} = m\mathbf{a}$ $\mathbf{F} = 5\times\{-8e^{-2t}\mathbf{i} + (6-6t)\mathbf{j}\}$ $= -40e^{-2t}\mathbf{i} + (30-30t)\mathbf{j}$	M1 A1	2	M1: Multiplying their acceleration by 5, even if not a vector. A1: Correct expression.
(ii)	Magnitude of **F** is $\{(-40)^2 + (30)^2\}^{\frac{1}{2}}$ $= 50$	M1 A1	2	M1: Finding magnitude from two non-zero terms. Must add terms and square root. Condone $\{(40)^2 + (30)^2\}^{\frac{1}{2}}$ A1: Correct answer only. In this part, condone lack of negative signs in expression for force in (b) (i).
(c)	When **F** acts due west, **j** component is zero $30 - 30t = 0$ $t = 1$	M1 A1	2	M1: Putting **j** component equal to zero. A1: Correct time.
(d)	$\mathbf{r} = -2e^{-2t}\mathbf{i} + (3t^2 - t^3)\mathbf{j} + \mathbf{c}$ When $t = 0$, $\mathbf{r} = 6\mathbf{i} + 5\mathbf{j}$ $\therefore$ $\mathbf{c} = 8\mathbf{i} + 5\mathbf{j}$ $\therefore$ $\mathbf{r} = (8 - 2e^{-2t})\mathbf{i} + (5 + 3t^2 - t^3)\mathbf{j}$	M1 A1 A1 dM1 A1	5	M1: Integration with either of the two components correct. Do not need to see **i** or **j**. A1: Correct **i** component. A1: Correct **j** component. Condone lack of + **c** dM1: Finding **c** using $6\mathbf{i} + 5\mathbf{j}$ and $e^0 = 1$. A1: Correct position vector.
	Total		**14**	

(Q3, June 2011)

16 (a)	using **F** = *m***a**: **a** = (6t – 1.2 t ^{2}) **i** + 2 e $^{-2t}$ **j**	M1 A1	2	ie dividing by 50
(b)	**v** = $\int$ **a** dt = (3 t^2 – 0.4 t^3) **i** – e^{-2t}**j** + **c** when t = 0, **r** = 7 **i** – 4 **j** **c** = 7 **i** – 3**j** **v** = (7 + 3 t^2 – 0.4 t^3) **i** – (3 + e $^{-2t}$) **j**	M1A1 m1A1	4	condone lack of + c; M1 one term correct ft from ke $^{-2t}$ in (b); just adding 7**i** – 4**j**, m0 accept unsimplified. CAO
(c)	when t = 1, **v** = 9.6 **i** – 3.135 **j** speed = $\sqrt{9.6^2 + 3.135^2}$ = 10.1 ms $^{-1}$	M1A1 m1 A1	4	ft from (b) ft from (b)
	Total		**10**	

(Q2, Jan 2012)

17 (a)(i)	$a = \frac{dv}{dt}$ $= 12t + 8e^{-4t}$ m s^{-2}	M1A1	2	M1 for either term correct
(ii)	When t = 0.5, a = $6 + 8 \times e^{-2}$ = 7.08 m s^{-2}	m1 A1	2	Condone 7.07 SC1 for 7.1 with no working
(b)	Using $F = ma$: $F = 4 \times 7.08$ = 28.3 N	B1ft	1	Ft from value awarded A1
(c)	$r = \int v\,dt$ $= 2t^3 + \frac{1}{2}e^{-4t} + 8t + c$ When $t = 0$, $r = 0 \rightarrow c = -\frac{1}{2}$ $r = 2t^3 + \frac{1}{2}e^{-4t} + 8t - \frac{1}{2}$	M1 A1 m1 A1	4	At least two terms correct Does not need $+c$ Does not need $c = -\frac{1}{2}$ Need r, s (or words)
	Total		**9**	

(Q2, June 2012)

18 (a)	Distance of particle from the origin is $\{(4\cos 3t)^2 + (4\sin 3t)^2\}^{\frac{1}{2}}$	M1		
	= 4 which is a constant ∴ particle is moving in a circle centre the origin	A1	2	
(b)	$\mathbf{v} = \frac{d\mathbf{r}}{dt}$ $\mathbf{v} = -12\sin 3t\,\mathbf{i} - 12\cos 3t\,\mathbf{j}$	M1A1	2	M1 for either term correct
(c)	$\mathbf{a} = \frac{d\mathbf{v}}{dt}$ $\mathbf{a} = -36\cos 3t\,\mathbf{i} + 36\sin 3t\,\mathbf{j}$	M1A1	2	M1 for either term correct
(d)	$\mathbf{a} = -9(4\cos 3t\,\mathbf{i} - 4\sin 3t\,\mathbf{j})$ $= -9\,\mathbf{r}$ $k = -9$	B2	2	B1 for 9
(e)	Acceleration is towards centre of circle (or origin)	E1	1	
	Total		**9**	

(Q4, June 2012)

19 (a)	$\mathbf{a} = \frac{d\mathbf{v}}{dt}$	M1		
	$= -4\pi\sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j}$	A1	2	M1 for either term correct Accept $-12\times\frac{\pi}{3}\sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j}$ condone no **i** in **(a)**
(b)(i)	Using $\mathbf{F} = m\mathbf{a}$: $\mathbf{F} = 4\times\left[-4\pi\sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j}\right]$	M1		Or either term correct
	$\mathbf{F} = -16\pi\sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 72t\mathbf{j}$	A1	2	
(ii)	When $t = 3$, $\mathbf{F} = 4\times[-4\pi\sin(\pi)\mathbf{i} - 54\mathbf{j}]$ $= -216\mathbf{j}$	B1		
	Magnitude is 216	B1ft	2	ft finding magnitude of their F
(c)	$\mathbf{r} = \int \mathbf{v}\,dt$	M1		either term correct
	$= \frac{36}{\pi}\sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 3t^3\mathbf{j} + \mathbf{c}$	A1		No need for **c** (otherwise cao) Condone $\frac{12}{(\pi/3)}$
	When $t = 3$, $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j}$	M1		
	$\rightarrow -81\mathbf{j} + \mathbf{c} = 4\mathbf{i} - 2\mathbf{j}$ $\mathbf{c} = 4\mathbf{i} + 79\mathbf{j}$	A1		
	$\mathbf{r} = \left\{\frac{36}{\pi}\sin\left(\frac{\pi}{3}t\right) + 4\right\}\mathbf{i} + \{79 - 3t^3\}\mathbf{j}$	A1	5	cao
	Total		**11**	*(Q2, Jan 2013)*

20 (a)	$v = \frac{ds}{dt}$	M1		
	$= 24t^2$	A1	2	
(b)	$a = \frac{dv}{dt}$			
	$= 48t$	B1		
	When $t = 2$, $a = 96$	B1		
	Using $F = ma$			
	$F = 3 \times 96$	M1		
	$= 288$ N	A1	4	
	Total		**6**	

(Q1, June 2013)

21 (a)	$v = \int a \, dt$			
	$= (20t^2 + t^3)\mathbf{i} - 5e^{-4t}\mathbf{j} + \mathbf{c}$	M1A1		M1 for either term correct Condone no '+ **c**'
	When $t = 1$,			
	$6\mathbf{i} - 5e^{-4}\mathbf{j} = 21\mathbf{i} - 5e^{-4}\mathbf{j} + \mathbf{c}$	M1		Finding '+ **c**'; not using $\mathbf{c} = 6\mathbf{i} - 5e^{-4}\mathbf{j}$
	$\mathbf{c} = -15\mathbf{i}$	A1		
	$\mathbf{v} = (20t^2 + t^3 - 15)\mathbf{i} - 5e^{-4t}\mathbf{j}$	A1	5	
(b)	When $t = 0$, $\mathbf{v} = -15\mathbf{i} - 5\mathbf{j}$	M1		
	Speed is $\sqrt{15^2 + 5^2}$	M1		
	$= 15.8$ m s^{-1}	A1	3	Accept $5\sqrt{10}$
	Total		**8**	

(Q3, June 2013)